SOLUTION OF EXERCISE # 1.2

Exercise # 1.2

Q.1: **Find the nature of the roots of the following equations:**

(i) $3x^2 + 7x - 2 = 0$

Sol. $3x^2 + 7x - 2 = 0$

Here $a = 3, \quad b = 7, \quad c = -2$

$\text{Disc} = b^2 - 4ac$

$= (7)^2 - 4(3)(-2) = 49 + 24 = 73$

Hence Roots are **Real, Irrational, Unequal**

(ii) $6x^2 = 7x + 5$

Sol. $6x^2 = 7x + 5$

$6x^2 - 7x - 5 = 0$

Here $a = 6, \quad b = -7, \quad c = -5$

$\text{Disc} = b^2 - 4ac$

$= (-7)^2 - 4(6)(-5) = 49 + 120 = 169$

Hence Roots are **Real, Rational, Unequal**

(iii) $2x^2 + 3x + 1 = 0$

Sol. $2x^2 + 3x + 1 = 0$

Here $a = 2, \quad b = 3, \quad c = 1$

$\text{Disc} = b^2 - 4ac$

$= (3)^2 - 4(2)(1) = 9 - 8 = 1$

Hence Roots are **Real, Rational, Unequal**

(iv) $\sqrt{2}x^2 + 3x - \sqrt{8} = 0$

Sol. $\sqrt{2}x^2 + 3x - \sqrt{8} = 0$

Here $a = \sqrt{2}, \quad b = 3, \quad c = -\sqrt{8}$

$\text{Disc} = b^2 - 4ac$

$= (3)^2 - 4(\sqrt{2})(-\sqrt{8}) = 9 + 4\sqrt{16} = 9 + 16 = 25$

Hence Roots are **Real, Rational, Unequal**

Q.2: **For what value of 'k' the roots of the given equations are equal.**

(i) $x^2 + 3(k+1)x + 4k + 5 = 0$ **(IIA-2019)**

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Sol. $x^2 + 3(k+1)x + 4k + 5 = 0$

Here $a = 1$, $b = 3(k+1)$, $c = 4k + 5$

As roots are equal, so Disc $= b^2 - 4ac = 0$

$(3(k+1))^2 - 4(1)(4k+5) = 0$

$9(k^2 + 2k + 1) - 16k - 20 = 0$

$9k^2 + 18k + 9 - 16k - 20 = 0$

$9k^2 + 2k - 11 = 0$

$9k^2 + 11k - 9k - 11 = 0$ by factorization

$k(9k + 11) - 1(9k + 11) = 0$

$(9k + 11)(k - 1) = 0$

Either OR

$9k + 11 = 0$ | $k - 1 = 0$

$9k = -11 \Rightarrow k = -11/9$ | $k = 1$

Hence $k = -11/9$ **or** $k = 1$

(ii) $\mathbf{x^2 + 2(k-2)x - 8k = 0}$

Sol. $x^2 + 2(k-2)x - 8k = 0$

Here $a = 1$, $b = 2(k-2)$, $c = -8k$

As roots are equal, so Disc $= b^2 - 4ac = 0$

$[2(k-2)]^2 - 4(1)(-8k) = 0$

$4(k^2 - 4k + 4) + 32k = 0$

$4k^2 - 16k + 16 + 32k = 0$

$4k^2 + 16k + 16 = 0$

$4(k^2 + 4k + 4) = 0$

$k^2 + 4k + 4 = 0$

$k^2 + 2k + 2k + 4 = 0$ by factorization

$k(k+2) + 2(k+2) = 0$

$(k+2)(k+2) = 0$

Either $k + 2 = 0$ OR $k + 2 = 0$

$k = -2$ | $k = -2$

Hence $k = 2$

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(iii) $(3k+6)x^2+6x+k=0$

Sol. $(3k+6)x^2+6x+k=0$

Here $a=3k+6,\quad b=6,\quad c=k$

As roots are equal, so

$\text{Disc}=b^2-4ac=0$

$(6)^2-4(3k+6)(k)=0$

$36-4k(3k+6)=0$

$36-12k^2-24k=0$

$-12k^2-24k+36=0$

$-12(k^2+2k-3)=0$

$k^2+2k-3=0$

$k^2+3k-k-3=0$ by factorization

$k(k+3)-1(k+3)=0$

$(k+3)(k-1)=0$

$k+3=0$ OR $k-1=0$

$k=-3$ | $k=1$

$\boxed{\text{Hence}\quad k=-3 \text{ or } k=1}$

(iv) $(k+2)x^2-2kx+k-1=0$

Sol. $(k+2)x^2-2kx+k-1=0$

Here $a=k+2,\quad b=-2k,\quad c=k-1$

As roots are equal, so

$\text{Disc}=b^2-4ac=0$

$\Rightarrow (-2k)^2-4(k+2)(k-1)=0$

$\Rightarrow 4k^2-4(k^2-k+2k-2)=0$

$\Rightarrow 4k^2-4(k^2+k-2)=0$

$\Rightarrow \cancel{4k^2}-\cancel{4k^2}-4k+8=0$

$\Rightarrow -4k+8=0$

$\Rightarrow -4k=-8$

$k=\frac{-8}{-4}=2$

$\boxed{\text{Hence}\quad k=2}$

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Q.3: Show that the roots of the equations:

(i) $a^2(mx+c)^2+b^2x^2=a^2b^2$ **will be equal if** $c^2=a^2m^2+b^2$

Sol. $a^2(mx+c)^2+b^2x^2=a^2b^2$

$$a^2\left[(mx)^2+2(mx)(c)+(c)^2\right]+b^2x^2-a^2b^2=0$$

$$a^2\left[m^2x^2+2cmx+c^2\right]+b^2x^2-a^2b^2=0$$

$$a^2m^2x^2+2a^2cmx+a^2c^2+b^2x^2-a^2b^2=0$$

$$a^2m^2x^2+b^2x^2+2a^2cmx+a^2c^2-a^2b^2=0$$

$$(a^2m^2+b^2)\,x^2+(2a^2cm)x+a^2c^2-a^2b^2=0$$

Here $A=(a^2m^2+b^2),\ B=(2a^2cm),\ C=(a^2c^2-a^2b^2)$

As roots are equal, so

$$\text{Disc}=b^2-4ac=0$$

$$(2a^2cm)^2-4(a^2m^2+b^2)\,(a^2c^2-a^2b^2)=0$$

$$4a^4c^2m^2-4(a^4c^2m^2-a^4b^2m^2+a^2b^2c^2-a^2b^4)=0$$

$$\cancel{4a^4c^2m}-\cancel{4a^4c^2m^2}+4a^4b^2m^2-4a^2b^2c^2+4a^2b^4=0$$

$$4a^4b^2m^2-4a^2b^2c^2+4a^2b^4=0$$

$$4a^2b^2\,(a^2m^2-c^2+b^2)=0$$

$$a^2m^2-c^2+b^2=0$$

$$a^2m^2+b^2=c^2$$

$\boxed{\text{Hence}\quad c^2=a^2m^2+b^2}$ **Proved.**

(ii) $(mx+c)^2=4ax$ **will be equal if** $c=\dfrac{a}{m}$ **(IIA-2017)**

Sol. $(mx+c)^2=4ax$

$$(mx)^2+2(mx)(c)+(c)^2=4ax$$

$$m^2x^2+2cmx+c^2-4ax=0$$

$$m^2x^2+2cmx-4ax+c^2=0$$

$$m^2x^2+(2cm-4a)\,x+c^2=0$$

Here $A=m^2,\quad B=2cm-4a,\quad C=c^2$

As roots are equal, so

$$\text{Disc}=B^2-4AC=0$$

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$(2cm-4a)^2-4(m^2)(c^2)=0$

$(2cm)^2+(4a)^2-2(2cm)(4a)-4c^2m^2=0$

$\cancel{4c^2m^2}+16a^2-16acm-\cancel{4c^2m^2}=0$

$16a^2-16acm=0$

$16a(a-cm)=0$

$a-cm=0$

$-cm=-a \Rightarrow c=\dfrac{-a}{-m} \Rightarrow \boxed{c=\dfrac{a}{m}}$ **Proved.**

(iii) $x^2+(mx+c)^2=a^2$ has equal roots if $c^2=a^2(1+m^2)$

Sol. $x^2+(mx+c)^2=a^2$

$x^2+(mx)^2+2(mx)(c)+(c)^2=a^2$

$x^2+m^2x^2+2cmx+c^2-a^2=0$

$x^2(1+m^2)+2cmx+c^2-a^2=0$

Here $A=(1+m^2)$, $B=2cm$, $C=c^2-a^2$

As roots are equal, so

$Disc=B^2-4AC=0$

$(2cm)^2-4(1+m^2)(c^2-a^2)=0$

$4c^2m^2-4(c^2-a^2+c^2m^2-a^2m^2)=0$

$\cancel{4c^2m^2}-4c^2+4a^2-\cancel{4c^2m^2}+4a^2m^2=0$

$4a^2+4a^2m^2-4c^2=0$

$4(a^2+a^2m^2-c^2)=0$

$a^2+a^2m^2-c^2=0$

$a^2(1+m^2)-c^2=0$

$-c^2=-a^2(1+m^2)$

$c^2=a^2(1+m^2)$

$\boxed{\text{Hence} \quad c^2=a^2(1+m^2)}$ **Proved.**

Q.4: If the roots of $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ are equal then, prove that: $a^3+b^3+c^3=3abc$.

SOLUTION OF EXERCISE # 1.2

Sol. $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$

Here $A = c^2 - ab$, $B = -2(a^2 - bc)$, $C = b^2 - ac$

As roots are equal, so

$\text{Disc} = B^2 - 4AC = 0$

$(-2(a^2 - bc))^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$4(a^4 + b^2c^2 - 2a^2bc) - 4(b^2c^2 - ac^3 - ab^3 + a^2bc) = 0$

$4a^4 + 4b^2c^2 - 8a^2bc - 4b^2c^2 + 4ac^3 + 4ab^3 - 4a^2bc = 0$

$4a^4 + 4ab^3 + 4ac^3 - 12a^2bc = 0$

$\Rightarrow 4a(a^3 + b^3 + c^3 - 3abc) = 0$

Either

$4a = 0$

$a = 0$

OR

$a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc$

$\boxed{a^3 + b^3 + c^3 = 3abc}$ **Proved**

Q.5: **Show that the roots of the following equations are real:**

(i) $x^2 - 2\left(m + \frac{1}{m}\right)x + 3 = 0$ **(IA-2019)**

Sol. $x^2 - 2\left(m + \frac{1}{m}\right)x + 3 = 0$

Here $a = 1$, $b = -2\left(m + \frac{1}{m}\right)$, $c = 3$

$\text{Disc} = b^2 - 4ac = \left[-2\left(m + \frac{1}{m}\right)\right]^2 - 4(1)(3)$

$= 4\left(m^2 + \frac{1}{m^2} + 2\right) - 12 = 4m^2 + \frac{4}{m^2} + 8 - 12$

$= 4m^2 + \frac{4}{m^2} - 4 = 4\left(m^2 + \frac{1}{m^2} - 1\right)$

Which is always positive for all values of 'm'

Hence roots are real. **Proved**

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(ii) $x^2 - 2ax + a^2 = b^2 + c^2$

Sol. $x^2 - 2ax + a^2 = b^2 + c^2$

$x^2 - 2ax + a^2 = b^2 + c^2$

$x^2 - 2ax + a^2 - b^2 - c^2 = 0$

Here $A = 1,\quad B = -2a,\quad C = a^2 - b^2 - c^2$

Disc $= B^2 - 4AC = (-2a)^2 - 4(1)(a^2 - b^2 - c^2)$

$= \cancel{4a^2} - \cancel{4a^2} + 4b^2 + 4c^2 \;= 4b^2 + 4c^2$

Which is always positive for all values of 'b' & 'c'

Hence roots are Real. **Proved**

(iii) $(b^2 - 4ac)x^2 + 4(a + c)x - 4 = 0$

Sol. $(b^2 - 4ac)\,x^2 + 4(a + c)\,x - 4 = 0$

Here $A = b^2 - 4ac,\quad B = 4(a + c),\quad C = -4$

Disc $= B^2 - 4AC = (4(a+c))^2 - 4(b^2 - 4ac)(-4)$

$= 16(a^2 + 2ac + c^2) + 16(b^2 - 4ac)$

$= 16a^2 + 32ac + 16c^2 + 16b^2 - 64ac$

$= 16a^2 - 32ac + 16c^2 + 16b^2$

$= 16[a^2 - 2ac + c^2 + b^2] \;= 16[(a - c)^2 + b^2]$

Which is always positive for all values of 'a', 'b' & c.

Hence roots are Real. **Proved**

Q.6: Show that the roots of the following equations are rational:

(i) $(a + 2b)x^2 + 2(a + b + c)x + (a + 2c) = 0$

Sol. $(a + 2b)x^2 + 2(a+b+c)x + (a+2c) = 0$

Here $A = a + 2b,\quad B = 2(a+b+c),\quad C = (a+2c)$

Disc $= B^2 - 4AC = [2(a+b+c)]^2 - 4(a+2b)(a+2c)$

$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ac) - 4(a^2 + 2ac + 2ab + 4bc)$

$= \cancel{4a^2} + 4b^2 + 4c^2 + \cancel{8ab} + 8bc + \cancel{8ac} - \cancel{4a^2} - \cancel{8ac} - \cancel{8ab} - 16bc$

$= 4b^2 + 4c^2 - 8bc$

$= 4(b^2 + c^2 - 2bc) = 4(b - c)^2 = [2(b - c)]^2$

Which is a perfect square.

Hence roots are Rational. **Proved**

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(ii) $(a+b)x^2 - ax - b = 0$

Sol. $(a + b)x^2 - ax - b = 0$

Here $A = a + b$, $B = -a$, $C = -b$

$$\text{Disc} = B^2 - 4AC$$
$$= (-a^2)^2 - 4(a+b)(-b)$$
$$= a^2 + 4ab + 4b^2$$
$$= (a + 2b)^2$$ Which is a perfect square,

Hence roots are Rational. **Proved**

(iii) $px^2 - (p-q)x - q = 0$

Sol. $Px^2 - (p - q)x - q = 0$

Here $a = p$, $b = -(p - q)$, $c = -q$

$$\text{Disc} = b^2 - 4ac = (-(p-q))^2 - 4(p)(-q)$$
$$= p^2 + q^2 - 2pq + 4pq$$
$$= p^2 + q^2 + 2pq$$
$$= (p+q)^2$$ Which is a perfect square,

Hence roots are Rational. **Proved**

Q.7: **For what value of 'k' the equation $(4-k)x^2 + 2(k+2)x + 8k + 1 = 0$ will be a perfect square.**

Sol. Here $a = 4-k$, $b = 2(k+2)$, $c = 8k+1$

As equation will be a perfect square, so roots are equal.

$$\text{Disc} = b^2 - 4ac = 0$$
$$(2(k+2))^2 - 4(4 - k)(8k + 1) = 0$$
$$4(k^2 + 4k + 4) - 4(32k + 4 - 8k^2 - k) = 0$$
$$4k^2 + 16k + 16 - 128k - 16 + 32k^2 + 4k = 0$$
$$36k^2 - 108k = 0$$
$$36k(k - 3) = 0$$

Either OR

$36k = 0 \Rightarrow k = 0$ | $k - 3 = 0 \Rightarrow k = 3$

Hence $\boxed{k = 0 \text{ or } k = 3}$